NISTIR 4740

Form Error Models of the NIST Algorithm Testing System

Mary Elizabeth A. Algeo
Theodore H. Hopp

U.S. DEPARTMENT OF COMMERCE
National Institute of
Standards and Technology
Manufacturing Engineering Laboratory
Factory Automation Systems Division
Machine Intelligence Group
Gaithersburg, MD 20899-0001

January, 1992

U.S. DEPARTMENT OF COMMERCE
Rockwell A. Schnavel,
Acting Secretary

NATIONAL INSTITUTE OF STANDARDS AND TECHNOLOGY
John W. Lyons, Director

Sponsored by
Navy Manufacturing Technology Program
and
National Institute of Standards and Technology

NISTIR 4740

Form Error Models of the NIST Algorithm Testing System

Mary Elizabeth A. Algeo
Theodore H. Hopp

Factory Automation Systems Division
National Institute of Standards and Technology
Gaithersburg, MD 20899-0001

January 1992

FORM ERROR MODELS OF THE NIST ALGORITHM TESTING SYSTEM

Mary Elizabeth A. Algeo and Theodore H. Hopp
National Institute of Standards and Technology
Gaithersburg, MD 20899-0001

The NIST Algorithm Testing System (or ATS) is a software package for testing the performance of geometric fitting software used in metrology systems[1]. One of the functions of the ATS is to generate test data simulating point measurements taken from manufactured parts. Data sets are generated by evaluating, according to user-specified sampling plans, models of parts with simulated form errors. The ATS also provides a simple model of point measurement errors. The purpose of this report is to document the form error models used in the ATS.

The ATS uses a parametric representation of geometry. A form error is simulated by computing, from the parametric coordinates of a point on an ideal geometric element, the cartesian coordinates of a form error vector at that point. This error vector is added to the ideal point vector to generate a point on a simulated surface with form error. A simulated measurement point is finally generated by adding a random measurement error vector. In summary, a point generated by the ATS data generator is of the form:

$$\mathbf{p}(u,v) = \mathbf{p}_0(u,v) + \Delta\mathbf{p}_F(u,v) + \Delta\mathbf{p}_M$$

where (u,v) are the parametric coordinates of the point (according to the sampling plan), $\mathbf{p}_0(u,v)$ is the point on the ideal geometry, $\Delta\mathbf{p}_F(u,v)$ is the form error, and $\Delta\mathbf{p}_M$ is the measurement error at that point. (The above description applies to surface geometries. Curve geometries are similar, but only use a single evaluation parameter.)

The ATS simulates different kinds of form errors for different ideal geometries. However, a random form error is supported for all curve and surface types. This consists of a displacement perpendicular to the ideal geometry by some random distance. Random form errors are not reproducible: evaluating the same surface at the same parametric coordinates will produce a different point. For other than random form errors, the ideal geometries and corresponding form error types are as follows:

Geometry	Form Error	Description
Line	Sine	A sinusoidal oscillation perpendicular to a line. The oscillation takes place in a plane determined by the ideal line definition.
	Step	A discontinuity in a line. After the step, the line continues in the same direction, but displaced from the original line. The displacement is in a plane determined by the ideal line definition.
	Bend	A change in direction of a line. At the bend, the line is continuous, but the direction of the line is discontinuous The new line lies in a plane determined by the ideal line definition.

[1]An overview report on the ATS is in preparation.

Geometry	Form Error	Description
Circle	Sine	A sinusoidal oscillation perpendicular to a circle, in the plane of the circle. To keep the curve simple, the amplitude of the oscillation should be less than the radius of the circle.
	Step	A discontinuity in the radius of the circle between two polar angles.
Plane	Sine	A two-dimensional sinusoidal oscillation about a plane. The oscillation is perpendicular to the plane.
	Step	A discontinuity in a plane at a line in the plane. One one side of the line of discontinuity, the plane is displaced, but parallel to, the plane on the other side.
	Bend	A sharp bend or crease in a plane along a line. On both sides of the line, the plane has local orthogonal coordinate systems. The normals are different.
Sphere	Sine	A two-dimensional sinusoidal oscillation perpendicular to a sphere. To keep the surface simple, the maximum amplitude of the oscillation should be less than the radius of the sphere.
Cylinder or Cone	Axial sine	A sinusoidal oscillation of the axis. The oscillation takes place in a plane containing the ideal axis. (Refer to the sine error for a line.)
	Radial sine	A sinusoidal oscillation of the radius at each cross section perpendicular to the axis. (Refer to the sine error for a circle.)
	Axial radius sine	A sinusoidal oscillation of the radius along the axis.
	Axial step	A discontinuity of the axis. (Refer to the step error for a line.)
	Radial step	A discontinuity in the radius at each cross section perpendicular to the axis. (Refer to the step error for a circle.)
	Axial radius step	A discontinuity in the radius along the axis.

The tables on the following pages summarize the form error models used in the ATS. There is one table for each type of ideal geometry supported by the ATS. Each table presents the types of form errors supported for that geometry, the parameters that define each error type, a name for each form error parameter, and an equation for the error vector as a function of parametric coordinates of points on the ideal surface.

Throughout these tables, the following special functions are used:

• The unit step function:

$$S(x) = \begin{cases} 0 & x < 0 \\ 1 & x \geq 0 \end{cases}$$

• The positive ramp function:

$$T(x) = \begin{cases} 0 & x \leq 0 \\ x & x > 0 \end{cases}$$

• The function U(α,ß): a uniformly-distributed random variate in the range [α,ß].

The tables also assume that each geometry has a local cartesian coordinate system, which is used to evaluate each form error vector. The local coordinate system is described briefly at the end of each table. The geometry of the ATS is based on STEP[2]. More complete information about geometry representations used in the ATS is in preparation.

[2] STEP is an acronym for an emerging international standard, the Standard for the Exchange of Product Model Data. The geometry of the ATS is based on Part 42 of STEP, specifically, the following document of ISO/TC 184/SC 4: *Product Data Representation and Exchange—Part 42: Integrated Resources: Geometric and Topological Representation*, ISO Committee Draft 10303-42, June 21, 1991.

LINES

Error	Parameter	Parameter name	Equation
Sine	A	amplitude	$A\sin(f\hat{u}+p)\,\mathbf{n}_\perp$
	f	frequency	
	p	phase	
Step	A	amplitude	$A\,\mathrm{S}(u-u_0)\,\mathbf{n}_\perp$
	u_0	location	
Bend	$\mathbf{n}_1$	new direction	$\mathrm{T}(u-u_0)(\mathbf{n}_1-\mathbf{n})$
	u_0	location	
Random	A	amplitude	$\mathrm{U}(-A,A)\,\mathbf{n}_\perp$

The local coordinate system consists of $\mathbf{n}$, the direction of the line.

In the above equations,

$$\mathbf{n}_\perp = \frac{\mathbf{n}' \times \mathbf{n}}{|\mathbf{n}' \times \mathbf{n}|}, \text{ where } \mathbf{n}' = \begin{cases} \mathbf{n}\times\mathbf{x} & |\mathbf{n}\cdot\mathbf{z}| > 0.9 \\ \mathbf{n}\times\mathbf{z} & |\mathbf{n}\cdot\mathbf{z}| \le 0.9 \end{cases}$$

is the unit direction perpendicular to $\mathbf{n}$ closest to the $+\mathbf{z}$ axis (or $+\mathbf{x}$ axis if $\mathbf{n}$ is close to $\mathbf{z}$). Also, if the line is parameterized from u_{min} to u_{max},

$$\hat{u} = 2\pi \frac{u-u_{min}}{u_{max}-u_{min}}$$

is the normalized coordinate of the point in the line segment being modeled. (In other words, the frequency f indicates the total number of wavelengths of form error oscillation in the line segment.)

CIRCLES

Error	Parameter	Parameter name	Equation
Sine	A	amplitude	$A\sin(fu+p)\,\mathbf{n}(u)$
	f	frequency	
	p	phase	
Step	A	amplitude	$A(\mathrm{S}(u-u_0)-\mathrm{S}(u-u_1))\,\mathbf{n}(u)$
	u_0	start angle	
	u_1	end angle	
Random	A	amplitude	$\mathrm{U}(-A,A)\mathbf{n}(u)$

The local coordinate system consists of $\mathbf{n}_1$ and $\mathbf{n}_2$, two perpendicular directions in the plane of the circle.

In the table,

$$\mathbf{n}(u) = (\cos u)\,\mathbf{n}_1 + (\sin u)\,\mathbf{n}_2$$

is the unit direction, away from the center, perpendicular to the circle at u.

PLANES

Error	Parameter	Parameter name	Equation
Sine	A_u	amplitude in u	$[A_u \sin(f_u \hat{u} + p_u) + A_v \sin(f_v \hat{v} + p_v)]\,\mathbf{n}_3$
	f_u	frequency in u	
	p_u	phase in u	
	A_v	amplitude in v	
	f_v	frequency in v	
	p_v	phase in v	
Step	A	amplitude	$A\,\mathrm{S}(u \sin\alpha - v\cos\alpha - d)\,\mathbf{n}_3$
	d	distance	
	α	orientation	
Bend	$\mathbf{n}_3'$	new normal	$\mathrm{T}(u\sin\alpha - v\cos\alpha - d)\cdot \lvert \mathbf{n}_3 - \mathbf{n}_3' \rvert \dfrac{(\mathbf{n}_3 + \mathbf{n}_3')}{\lvert \mathbf{n}_3 + \mathbf{n}_3' \rvert}$
	d	distance	
	α	orientation	
Random	A	amplitude	$\mathrm{U}(-A, A)\mathbf{n}_3$

The local coordinate system consists of $\mathbf{n}_1$, $\mathbf{n}_2$, and $\mathbf{n}_3$, three mutually perpendicular unit vectors, with $\mathbf{n}_3$ perpendicular to the plane.

Also, if the plane is parameterized from u_{min} to u_{max} in u, and similarly for v,

$$\hat{u} = 2\pi \frac{u - u_{min}}{u_{max} - u_{min}}$$

$$\hat{v} = 2\pi \frac{v - v_{min}}{v_{max} - v_{min}}$$

are the normalized coordinates of the point within the planar patch being modeled. (In other words, the frequencies f_u and f_v indicate the total number of wavelengths of form error oscillation in the u and v extents.)

SPHERES

Error	Parameter	Parameter name	Equation
Sine	A_u	amplitude in u	$[A_u \sin(f_u u + p_u) + A_v \sin(f_v v + p_v)]\,\mathbf{n}(u,v)$
	f_u	frequency in u	
	p_u	phase in u	
	A_v	amplitude in v	
	f_v	frequency in v	
	p_v	phase in v	
Random	A	amplitude	$\mathrm{U}(-A,A)\mathbf{n}(u,v)$

The local coordinate system consists of $\mathbf{n}_1$, $\mathbf{n}_2$, and $\mathbf{n}_3$, three mutually perpendicular unit vectors.

In the above equations:

$$\mathbf{n}(u,v) = (\cos u)(\sin v)\,\mathbf{n}_1 + (\sin u)(\sin v)\,\mathbf{n}_2 + (\cos v)\,\mathbf{n}_3$$

is the unit direction, away from the center, perpendicular to the sphere at (u,v).

CYLINDERS and CONES

Error	Parameter	Parameter name	Equation
Axial Sine	A	amplitude	$A\sin(f\hat{v}+p)\mathbf{n}(u_0)$
	f	frequency	
	p	phase	
	u_0	reference angle	
Radial Sine	A	amplitude	$A\sin(fu+p)\mathbf{n}(u)$
	f	frequency	
	p	phase	
Axial Radius Sine	A	amplitude	$A\sin(f\hat{v}+p)\mathbf{n}(u)$
	f	frequency	
	p	phase	
Axial Step	A	amplitude	$A\,\mathrm{S}(v-v_0)\,\mathbf{n}(u_0)$
	v_0	location	
	u_0	reference angle	
Radial Step	A	amplitude	$A(\mathrm{S}(u-u_0)-\mathrm{S}(u-u_1))\,\mathbf{n}(u)$
	u_0	start angle	
	u_1	end angle	
Axial Radius Step	A	amplitude	$A\,\mathrm{S}(v-v_0)\,\mathbf{n}(u)$
	v_0	location	
Random	A	amplitude	$\mathrm{U}(-A,A)\,\mathbf{n}(u)$

The local coordinate system consists of $\mathbf{n}_1$, $\mathbf{n}_2$, and $\mathbf{n}_3$, three mutually perpendicular unit vectors, with $\mathbf{n}_3$ directed along the axis of the cylinder or cone.

In the above equations,

$$\mathbf{n}(u) = (\cos u)\,\mathbf{n}_1 + (\sin u)\,\mathbf{n}_2$$

is the unit direction, away from the axis, perpendicular to the axis at u (for all v). Note that, in the case of a cone, the form error is not perpendicular to the surface.

Also, if the cone is parameterized from v_{min} to v_{max} in v (axially),

$$\hat{v} = 2\pi \frac{v - v_{min}}{v_{max} - v_{min}}$$

is the normalized v coordinate of the point within the axial segment being modeled. (In other words, the frequency f indicates the total number of wavelengths of form error oscillation along the axis.)